Collateralization and asset price bubbles when investors disagree about risk

Tobias Broer, Paris School of Economics,
IIES, Stockholm University and CEPR*

Afroditi Kero, Neapolis University, Pafos, Cyprus.

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Abstract

Survey respondents disagree strongly about the dispersion of future returns and, increasingly, macroeconomic uncertainty. Such disagreement about risk may raise asset prices when collateralized debt products allow investors to realize perceived gains from trade. Investors who expect low volatility in collateral cash-flow appreciate senior debt as riskless. Those who expect high volatility, in contrast, value the upside potential in junior debt or equity claims. We show how such self-selection may have had a sizeable effect on the prices of RMBS and CDOs before the crisis, as investors disagreed about the volatility of aggregate economic conditions and their importance for default rates in collateral pools.

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Keywords: Asset Prices, Heterogeneous Beliefs, Disagreement, Volatility, Securitization, Structured Finance, Bubbles

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*Corresponding author: Tobias Broer; Adress: IIES, Stockholm University, 10691 Stockholm, Sweden; Tel: +46 8 16 30 56; Fax: +46 8 16 14 43; Email: tobias.broer@iies.su.se. A previous version of this paper was entitled "Collateralization bubbles when investors disagree about risk". We thank Jaakko Meriläinen, John Kramer, Philipp Hochmuth and Fabian Sinn for more than outstanding research assistance. We also thank Nick Bloom, Markus Brunnermeier, Piero Gottardi, John Geanakoplos, Jungsuk Han, Alex Kohlhas, Per Krusell, Alex Michaelides, Alp Simsek and Dimitri Vayanos, as well as seminar participants at EIEF, Institute for Advanced Studies in Vienna, EUI Alumni Workshop 2013, European Summer Symposium in Economic Theory 2013, ESSIM 2014, the Greater Stockholm Macro Group, 13th Conference on Research on Economic Theory and Econometrics, University of Edinburgh and University of Oslo, for helpful comments. Broer gratefully acknowledges financial support from the European Research Council under Advanced Grant 230574.

1 Introduction

From the mid-1990s to the beginning of the Great Recession, the world economy has seen an unprecedented wave of financial innovation, partly in the form of new collateralized debt products. At the same time, the prices of collateral assets, such as real estate, but also stocks, experienced an unprecedented increase. This paper links these two phenomena to a third, less documented one: disagreement among investors about economic risk. We provide evidence for this argument from several US surveys. We first show how the data analyzed by Amromin and Sharpe (2008) and Ben-David et al. (2013) imply strong disagreement among both retail investors and finance professionals about the dispersion of future stock returns. Second, to analyze a longer time horizon covering the Great Moderation period, we document that, since the early 1990s, near-term GDP forecasts from the Survey of Professional Forecasters show increasing disagreement among forecasters about the dispersion of GDP growth, while disagreement about mean growth has fallen. We conclude from this that disagreement about risk is substantial. And there is some evidence that suggests that it became more important relative to disagreement about mean payoffs in the 1990s and early 2000s.

We show how such heterogeneous risk perceptions, when combined with financial innovation in the form of collateralized debt products, can create asset price bubbles. In the absence of collateralization, risk-neutral investors trade assets at their common fundamental value even if they disagree about payoff risk. The introduction of risky collateralized debt products increases asset prices above this common fundamental value by splitting the cash-flow into senior debt and junior debt or equity claims. Investors who perceive low volatility are happy to pay high prices for senior debt, which they regard as riskless. Those who think that volatility is high, in contrast, value the upside potential in junior claims. Disagreement about risk thus raises the equilibrium price of collateral assets as investors self-select into buying the claims they value most highly. We show how this may have been a factor behind the boom in 'Structured Finance' assets, such as collateral debt obligations (CDOs), whose senior tranches are attractive to investors who believe in diversification and thus think that the default rates of collateral pools are stable.

Those, in contrast, who think that default rates are more reflective of aggregate conditions, and thus more volatile, think that senior tranches may still fail in bad times, but are happy to pay for junior and equity tranches, which they expect to pay when conditions are sufficiently good.

Our simple theoretical benchmark model focuses on a given mortgage pool whose cash-flow can be traded by risk-neutral investors in the form of a debt and an equity tranche. The insight that collateralization increases asset prices, however, applies to any risky debt contract collateralized by a payoff about whose dispersion investors disagree. In a quantitative application of our theory, we consider more complex debt instruments that split collateral cash-flow into 'tranches' that receive payments in strict order of their pre-specified seniority. Such structured securities had experienced a spectacular boom during the early 2000s that came to an abrupt end with the financial crisis. We find that even modest disagreement about the variability of default rates can raise the market value of a typical US residential mortgage backed security (RMBS) by more than 100 basis points above the expectation of collateral cash-flow (that we assume is shared by all investors). Disagreement about risk may thus be an additional reason for the boom in Structured Finance during the early 2000s,[1] although the precise timing of that boom is likely explained by other factors pointed out in the literature.[2]

Structured Finance is not the only asset class where our theory may be important. For example, our results have implications for the theory of firm financing: they call for a mix of debt and equity finance that depends on the heterogeneity of risk perceptions in the investor

[1]Other factors that contribute to the attractiveness of structured debt are the mitigation of information asymmetries and the creation of safe assets through issuance of (super-) senior tranches, regulatory arbitrage (Acharya et al., 2013; Brunnermeier, 2009), rating bias (Griffin and Tang, 2011; 2012), as well as investors' disregard of certain unlikely risks (Gennaioli et al., 2012) or of their highly systemic nature (Coval et al., 2009b).

[2]Such factors include financial innovation and deregulation in the US that made it easier to collateralize large baskets of mortgage loans and other risky assets (Boz and Mendoza, 2014), the advent of a large pool of standardized high-risk collateral in the form of US subprime or Alt-A mortgages (attributed for example to the technological innovation in underwriting procedures, Gorton (2009), Gates et al. (2002)), the disintermediation of the US financial system (boosting the demand for repo collateral), changes in banking regulation (reducing the relative capital requirements for investments in senior securitization tranches), the low interest rate environment of the early 2000s, or the boost to the private-label RMBS market through the sale of mortgage portfolios by the US government after the savings and loans crisis of the late 1980s / early 1990s. The role of affordable-housing policy in the subprime boom, in contrast, is controversial. On these points see e.g. the Financial Crisis Inquiry Commission's Report (Financial Crisis Inquiry Commission, 2011, p. 68–80).

pool. Specifically, firms optimally issue debt to investors who perceive risk to be low, and sell equity to those who perceive higher risk and thus stronger upside potential for shares in the firm.

Previous studies of disagreement have largely focused on disagreement about an asset's mean payoff, where 'optimists' expect higher payoffs than 'pessimists' and, absent short-selling, drive prices above average valuations (Miller, 1977). Leverage through riskless collateralized loans may raise prices further by increasing investment funds of optimists (Geanakoplos, 2003). When risky collateralized debt can be issued (Simsek, 2013), optimists face a trade-off: in order to raise funds for investment into upside risk, they have to sell downside risk at unfavorable prices to pessimists. With trade in collateralised debt contracts, the effect of disagreement about mean payoffs is thus dampened when optimists have positive views mainly about downside risk. Importantly, the asset price does not exceed optimist valuations unless more complex assets are traded or investors are borrowing-constrained (Fostel and Geanakoplos, 2008; 2012).

Disagreement about the dispersion of payoffs affects the price of collateral assets in a way that is fundamentally different. Investors who perceive asset payoffs to be more volatile than others are optimistic about upside risk at the same time as they are pessimistic about downside risk. Conversely, low-volatility investors are downside optimists and upside pessimists. By allowing them to trade up- and downside risk separately, risky collateralised debt leads to self-selection of investors into buying their preferred risks. This realizes pure gains from trade and raises prices above the *maximum* valuation of collateral assets. This is in contrast to disagreement about means, as e.g. in Simsek (2013), where the optimistic valuation is typically an upper bound for the asset price. Relative to Simsek (2013), we also provide explicit conditions in terms of exogenous variables for an increase in disagreement to increase asset prices further. Simsek's (2013) Theorem 5, in contrast, states conditions that involve the endogenous face value $\bar{s}$. Similar to Example 2 in Simsek (2013), our Example 1 illustrates that an increase in disagreement may indeed also lower asset prices. Geerolf (2018) considers a continuum of investors in an environment similar to that of Simsek (2013) but with point beliefs and shows how this implies, in equilibrium, a bilateral assignment of lenders and borrowers to collateralised loan contracts that differ in interest and face value.

Because we interpret the collateral asset as a pool of loans whose defaults investors perceive as more or less correlated, our theoretical analysis contributes to a recent literature that studies the effect of disagreement about default characteristics on prices of collateralized debt tranches. In particular, Broer (2018) discusses qualitatively the effects of disagreement about average default rates and their variability on the prices of structured finance assets using a simple two-loan example. The theoretical analysis of this paper studies disagreement about risk in a substantially richer environment with many collateral assets. Moreover, we also identify a sufficient condition for an increase in disagreement to increase the collateral price further, and provide an example where it reduces the collateral price. After our main analysis had been complete, Ellis et al. (2017) showed that structuring in tranches is the optimal security design within the set of monotone securities for general disagreement. They show that there exists a unique tranching equilibrium, and, like the theoretical part of this paper, provide conditions such that collateral prices exceed the valuations of all investors. Their environment, with a dedicated issuer choosing an optimal number of tranches, differs from that of our theoretical analysis with a simple two-tranche structure that makes the results immediately applicable to any setting where loans are collateralized by any risky asset. Moreover, and related to our empirical evidence, we discuss when an increase in disagreement about risk raises collateral prices (and provide a simple, if extreme, example where it lowers them).[3] In related work, Gong et al. (2020) consider a general equilibrium model with heterogeneous investors and collateralised borrowing and show that when a risky asset can be tranched or when derivative contracts backed by the asset can be used as collateral for other assets ("pyramiding"), the equilibrium risk premium of the asset may be smaller than the price of insuring its risk (implying positive "basis").

Since the equity and debt tranches we consider are equivalent to, respectively, buying a call option and selling a put option on the payoff of the underlying loan portfolio, our analysis

[3]Bianchi and Jehiel (2016) consider disagreement about average default probabilities in a similar context.

also relates to the literature on the effects of heterogeneous beliefs on options prices (Li, 2007; 2013; Osambela, 2015; Feng et al., 2015). Perhaps because volatility is perfectly observable for continuous-time Brownian motion, that literature abstracts from disagreement about volatility, while we focus on contexts where either sampling or the underlying shock process are discrete, which we think is particularly true for innovations originating from macroeconomic shocks.

The next section presents evidence from US surveys that shows disagreement about return risk to be important, and disagreement about macroeconomic risks to have increased between 1990 and 2016, when it accounted for about half of overall disagreement. Section III presents a simple general-equilibrium model with two investor types, whose disagreement about risk leads to an asset-price bubble when they can trade collateralized debt whose riskiness is determined endogenously. Section IV uses a quantitative model of structured loan pools to gauge the effect of disagreement about credit risk for the US mortgage market.

2 Motivating evidence: Disagreement about risk in US surveys

One aim of this paper is to show how a reasonable amount of disagreement about risk can have an important impact on asset prices. This section provides evidence about the importance of such disagreement. We concentrate on respondents that are incentivized to have good information (investors with substantial stock investments (in the Michigan survey), financial executives (Ben-David et al. (2013)), professional forecasters), and thus interpret heterogeneity in the dispersion of reported distributions as disagreement about risk, and not (rational) ignorance or agnosticism.

2.1 Disagreement about US stock market returns

This section uses information from two US surveys to show how investors strongly disagree not only about expected returns, but also about return risks. Table 1 reports summary statistics of the supplementary questions in the Michigan Survey of Consumer Sentiments, covering 22 surveys in the years 2000 to 2005, taken from Amromin and Sharpe (2008).[4] Expected annual returns are on average close to the realised average return of 10 percent during the period, but widely dispersed: 10 percent of respondents expect an average return of below 5 and above 16 percent respectively . The perceived riskiness of stock investments, however, also varies strongly across investors: while 10 percent of respondents believe realized returns to fall within 2 percentage points of their expectation with a probability of at least 80 percent, another 10 percent expect returns to fall <u>outside</u> this range with at least 80 percent probability.[5] Importantly for our analysis, these differences in the expected dispersion of the stock market can be interpreted as disagreement about the correlation of individual assets within the index, similar to our quantitative analysis in Section 4.[6]

Table 1: Return Expectations in the Michigan Survey 2000-2005

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	N	Mean	10th pct	25th pct	Median	75th pct	90th pct
Expected return R_e	3,046	10.4	5	7	10	12	16
Prob $\|R - R_e\| < 2pp$	3,015	43.3	20	25	50	50	80
Implied σ_{10-20} (in percent)	2,854	4.56	1.56	1.73	2.96	2.96	7.88

The first row reports the distribution of investors' answer to the question about the "annual rate of return that you would expect a broadly diversified portfolio of U.S. stocks to earn, on average". The second row reports the probability "that the average return over the next 10 to 20 years will be within two percentage points of your guess", and the third one shows the corresponding standard deviation assuming normally distributed beliefs about stock market returns.

Ben-David et al. (2013) present similar survey evidence for a sample of senior finance executives, mainly Chief Financial Officers. They show how their respondents' forecasts of US S&P 500 returns are 'miscalibrated', in the sense that respondents underestimate the uncertainty around their expected returns both relative to history and relative to subsequent outcomes.

[4]The authors eliminate incomplete responses, those deemed by the interviewer to have a low "level of understanding" or a poor "attitude" towards the survey, and those that answered "50 percent" to all probability questions.

[5]The question asks for the probability "that the average return over the next 10 to 20 years will be within two percentage points of your guess". We interpret the heterogeneity in responses to this question as evidence of heterogeneous perceptions of risk. An alternative interpretation is that of heterogeneous confidence in individual point estimates.

[6]Using a normality assumption to transform these assessments into standard deviations, the 90-10 percentile difference of standard deviations equals 6.3, compared to 11 for expected returns.

Interestingly for the present study, they also show how respondents strongly disagree in their volatility estimates. In particular, the individual standard deviations of 1-year-return forecasts implied by their survey responses have a distribution whose 95-5 percentile difference equals 15 percentage points (both for the whole sample from 2001 to 2011, and the 2011Q1 cross section).

2.2 Disagreement about US Macro Risk 1991-2016

The Survey of Professional Forecasters (SPF) is a quarterly anonymous survey that asks forecasters to indicate, among other measures, their probability distribution for GDP growth in the current calendar year.[7] Based on a normal approximation of individual forecast distributions, as in Giordani and Söderlind (2003), we calculate time series of forecaster-specific means μ_{it} and standard deviations σ_{it}, where i denotes forecasters and t the forecast period.[8] The top left panel of Figure 1 shows that the dispersion of σ_{it} across forecasters (as indicated by the cross-sectional standard deviation) rose throughout the 1990s, and again after the beginning of the financial crisis in 2007. In contrast, the dispersion of mean forecasts (in the top right panel) showed no such rise, but rather declined persistently after its spike following the Great Recession.

To assess the relative importance of heterogeneous means and standard deviations for overall forecaster disagreement, the remaining panels of Figure (1) look at a measure of total disagreement based on the integral of absolute differences of any two forecaster-specific normal densities f_i, f_j

$$d = \frac{1}{2}\frac{1}{N_t^2}\sum_i\sum_j \int |f_i(g_y) - f_j(g_y)| dg_y, \tag{1}$$

[7] We limit our sample to the years 1991 to 2019. The sample size of the SPF had shrunk to only 15 forecasters before the survey was taken over by the Federal Reserve Bank of Philadelphia during the course of 1990 and its coverage increased to around 45, which we deem too low for studying cross-sectional moments. Disagreement measures in the 1980s were indeed extremely volatile, as shown in Broer and Kero (2014), where we had overlooked the small sample sizes during that period. To keep the forecasting horizon constant and equal to the remainder of the current year, we only use data collected during the first quarter of every year.

[8] The online appendix shows that for the disagreement measures that we can calculate without this interpolation, the results from using the reported histograms directly are extremely similar.

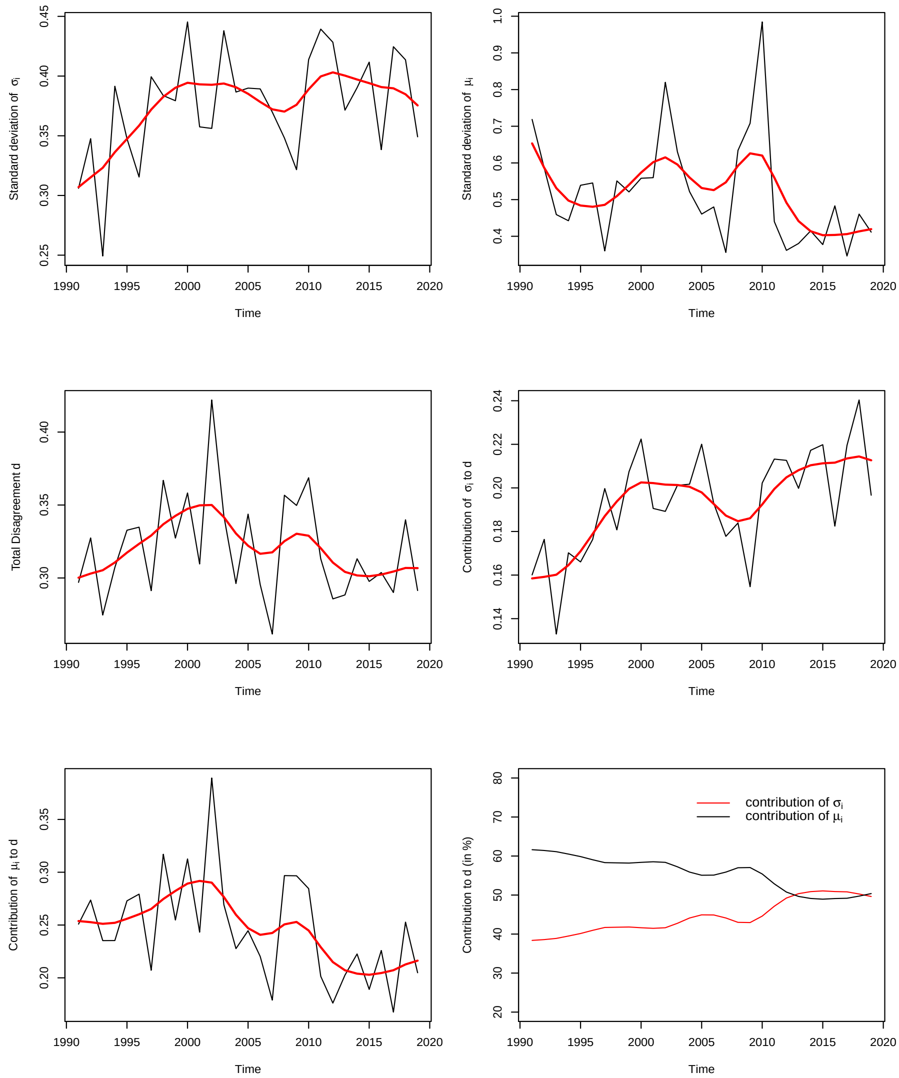


Figure 1: The top-left panel plots the time series of the cross-sectional standard deviation of σ_{it} (the standard deviation of forecaster i's reported distribution of GDP growth in the currrent calendar year) in the SPF, derived using a normal approximation to the forecast distribution as in Giordani and Söderlind (2003). The top-right panel plots the standard deviation of means μ_{it}. The remaining four panels show the total disagreement measure d (center-left panel), the contribution of heterogeneous forecast standard deviations σ_{it} (center-right panel), and of heterogeneous forecast means (bottom-left) panels, as well as the percentage of disagreement accounted for by those two parameters (bottom-right panel). The red lines in the first 5 panels show the trend from an HP filter with smoothing parameter 25 (to adjust for the annual frequency, see Ravn and Uhlig (2002)). We omit two observations at the beginning and end of the sample to reflect the two-sided nature of the filter.

where N_t is the time-varying number of forecasters in the sample.[9] This measure allows us to calculate the contribution of the heterogeneity in standard deviations to overall disagreement using the formula in (1) with the mean of the two normal distributions held constant ($\mu_{it} = \mu_{jt}$), and equivalently for the contribution of heterogeneity in means. Overall forecaster disagreement, as measured by d in Equation (1), (depicted in the center-left panel) fluctuated around a roughly inverse-U shaped pattern. This masks opposing trends in the contribution of heterogeneous standard deviations (strongly rising in the center-right panel) and that of heterogeneous means (which has fallen substantially from its peak in 2000, in the bottom-left panel). The percentage of overall disagreement that can be attributed to disagreement about forecast dispersion rose throughout most of the sample period, and has been similar to that attributable to disagreement about mean growth since 2013 (bottom-right panel).[10]

Our conclusion from the evidence presented in this section is twofold. First, perhaps unsurprisingly, there is strong disagreement in US surveys about the dispersion of asset returns and macroeconomic outcomes. Second, disagreement about GDP growth risk in the SPF increases in the 1990s and 2000s both in absolute terms and, in particular, relative to disagreement about mean growth. This is interesting in the context of heterogeneous perceptions of financial risks because fluctuations in aggregate output growth are an important determinant of asset payoffs in many contexts. More importantly, it is precisely this disagreement about the (relative) importance of macro sources of volatility, as opposed to diversifiable sources at the creditor or regional level, that translates into disagreement about the co-movement of default events when collateral pools consist of more than one asset: the more volatile an investor thinks aggregate factors are relative to idiosyncratic ones, the more correlated she expects defaults to be. In other words, disagreement about macro volatility translates into disagreement about default correlation among loans in a collateral pool. The next section studies the consequences of such disagreement for asset prices.

[9]This measure equals zero for any two identical distributions and is bounded above by 1 (for two disjoint distributions).

[10]Note that the two contributions do not exactly sum to total disagreement d.

3 Theory: How disagreement about risk can lead to a bubble in collateral prices

This section studies a simple equilibrium economy where two risk-neutral investor types disagree about the riskiness, but not the expected value, of payoffs from an asset (or a pool of collateral loans) that they can buy and use as collateral for issuing debt. We show two results: first, self-selection of investors into holding, respectively, a senior debt tranche and a junior equity tranche raises the equilibrium price of the mortgage pool above its common expected payoff. Second, we derive conditions for an increase in disagreement to increase the equilibrium price further. We also compare our main results to the case of disagreement about mean payoffs.[11]

3.1 The general environment

We study an economy that exists for two periods $t \in \{0, 1\}$, with two types of agents $i \in \{H, L\}$, both of unit mass. In period 0, agents of type i receive an endowment $n_i > 0$ of the unique perishable consumption good and 1 unit of a risky portfolio that contains a large number of risky assets, indexed by l. We call these assets 'mortgages', but they could be any other kind of risky claims. Mortgages pay an exogenous stochastic amount s_l in period 1 that is bounded by a recovery value V_{rec} below, and by their face value 1 above. For concreteness, assume $s_l = E^s + (1-\theta)\varepsilon_l + \theta\varepsilon$ where $E^s < 1$ is a positive constant, ε_l a loan-specific and ε a common random shock. ε_l and ε follow independent and continuous distributions on a finite support $\mathbb{E} = [V_{rec} - E^s, 1 - E^s]$ with 0 mean. In particular, the distribution of ε is described by a cumulative distribution function F_ε that is continuous and strictly increasing on its support $[\underline{\varepsilon}, \bar{\varepsilon}] \subseteq \mathbb{E}$. Assuming that idiosyncratic shocks $\varepsilon_l, \varepsilon_k, \forall l \neq k$ are independent, the mortgage portfolio pays $s = \int s_l dl = E^s + \theta\varepsilon$ by the law of large numbers.

[11]The focus on trade in two assets, senior debt and junior equity, is without loss of generalisation in our simple setting with only two investor types (whose perceived cash-flow distributions we assume only cross once). Section 4 considers more complex assets with several investor types. A previous working paper version of this article (Broer and Kero, 2014) presents results for the general case with a continuum of types.

Agents agree about the mean of payoffs E^s but disagree about their dispersion. For this purpose, we normalise the distribution of ε to be common across investors, but allow heterogeneity in perceptions of θ, which can be interpreted as the perceived relative variability of aggregate vs idiosyncratic determinants of loan defaults, and determines the perceived variance of payoffs from the mortgage pool. Specifically, we assume that type L and type H investors perceive θ to equal θ_L and θ_H, respectively, with $\theta_L < \theta_H$. Relative to the 'low-risk' type L, the 'high-risk' type H thus believes that the payoff variance of the mortgage portfolio is higher, and that payoffs are therefore less tightly distributed.

We denote the cumulative distribution function of s perceived by type i as $F_i : S \longrightarrow R^+$, with f_i being the corresponding pdf. Our assumptions imply that F_L strictly second-order dominates F_H, which it crosses exactly once at $\varepsilon = 0$. The main results in this section continue to hold under the more general assumption that beliefs about payoffs satisfy second-order stochastic dominance with a common mean.

In our prefered interpretation of this environment, heterogeneous risk perceptions arise from contrasting views about the importance of aggregate vs. idiosyncratic factors in determining mortgage defaults. Type L investors who think that loan-specific factors are the dominant source of defaults, and thus think that diversification through pooling can eliminate most risk, expect the pool's payoff to be tightly distributed around its mean and regard it as high-quality collateral for debt. At the same time they see little upside potential in a leveraged pool's payoff after the debt it collateralizes has been paid. Type H investors, in contrast, who think that loan risk comes mostly from aggregate shocks, or who perceive these to be more volatile, believe in volatile payoffs. They thus expect collateralized debt to default in bad times but equity tranches to pay when times are good. Self-selection of investors then raises the prices of both collateralized debt and of the collateral pool. An equally valid interpretation of our framework, however, is one where investors leverage purchases of a single risky asset using collateralized debt.

All agents are risk-neutral with preferences $U_i = c_i + \frac{1}{R}E_i(c_i')$, where E_i is the mathematical expectation of agent i, c_i (resp. c_i') denotes consumption in period 0 (resp. 1) and $\frac{1}{R} \leq 1$ is the discount factor. At the end of $t = 0$, agents trade the mortgage pool at unit price p. While we assume that agents cannot trade uncollateralized claims (for example because there is no commitment to repayments in the final period 1) they can use the cash-flow from the mortgage pool as collateral for structured debt securities. We concentrate on the simplest form of these securities, which allocate the cash-flow from the mortgage pool to a debt and equity tranche (but consider more realistic, complex structures in Section 4). The debt tranche is a senior claim on the collateral cash-flow that promises to pay a face value $\bar{s}$ or the payoff of the loans that serve as collateral, whatever is smaller. Normalizing contracts to have 1 unit of the portfolio as collateral,[12] the debt tranche thus has unit payoffs equal to $\min\{s, \bar{s}\}$, which are trivially concave in s. The equity tranche simply pays the remainder $\max\{0, s - \bar{s}\}$, which is convex in s. Because buying the mortgage pool and selling - or issuing - the debt tranche is payoff-equivalent to buying the equity tranche, there is an obvious multiplicity in portfolios. In the following, we therefore concentrate on trade in the mortgage portfolio, possibly leveraged by issuing debt tranches that trade at price $q(\bar{s})$, which we call 'collateralized debt' for simplicity. These debt contracts must fulfill the collateral constraint

$$\int_0^1 b_i(\bar{s}) d\bar{s} \geq -a_i \tag{2}$$

where $b_i(\bar{s})$ are agent i's holdings of collateralized debt contracts with face value $\bar{s}$, and $a_i > 0$ denotes her holdings of the mortgage pool.

The budget constraints of agent i in $t = 0$ and $t = 1$, respectively, are:

$$c_i + pa_i + \int_0^1 q(\bar{s}) b_i(\bar{s}) d\bar{s} \leq n_i + p, \tag{3}$$

$$c_i' \leq a_i s + \int_0^1 min\{s, \bar{s}\} b_i(\bar{s}) d\bar{s}, \tag{4}$$

Figure 2 illustrates the (gross) unit profits of different investments as a function of the mortgage

[12]Note that one unit of the debt tranche collateralized by x units of the pool is payoff-equivalent to x

Unit Profits

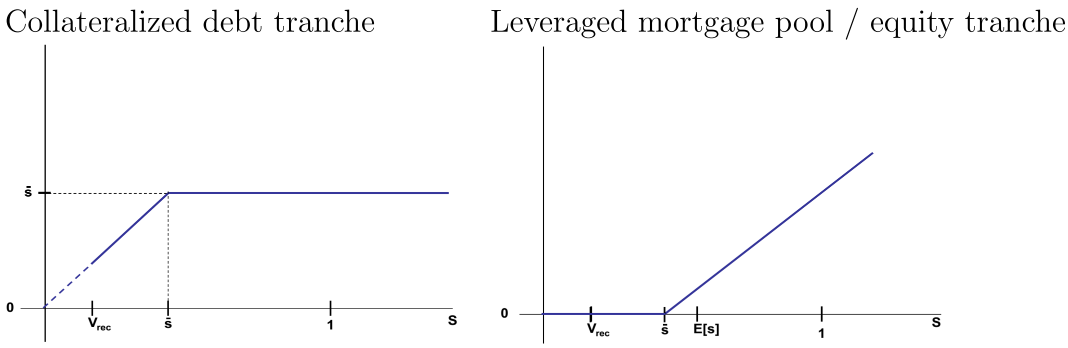


Figure 2: For a given face value of debt $\bar{s}$, the left panel plots the profits from collateralized debt. The right panel plots those from purchases of the equity tranche, or from leveraging the mortgage portfolio by issuing collateralized debt.

pool's payoff s, at given face value $\bar{s}$. While the profits from a unit of collateralized debt (left panel), equal to $\frac{[min\{s,\bar{s}\}]}{R}$, are concave in s, those from a mortgage portfolio fully leveraged by issuing an equal amount of units of debt with face value $\bar{s}$ (right panel) are $\frac{s-\min(s,\bar{s}))}{R} = \frac{\max(0,s-\bar{s}))}{R}$ and thus convex in s. Given the second-order stochastic dominance relationship of beliefs, this immediately implies that investors with more dispersed beliefs expect to make higher profits from investment in the leveraged mortgage portfolio, denoted as Π_i^a. Investors with less dispersed beliefs expect higher profits from debt tranches Π_i^d.

Lemma 1 - *Profits and risk perceptions*

Consider a given face value of debt contracts $\bar{s}$. Type L investors expect higher profits from investing in risky collateralized debt than type H investors. The inverse is true for profits from the leveraged mortgage portfolio:

$$\Pi_H^d = E_H\left[\frac{[min\{s,\bar{s}\}]}{R}\right] - q(\bar{s}) \leq \Pi_L^d,\ \forall\ \bar{s} \in (V_{rec}, 1), \forall p, q(\bar{s}), R,$$

$$\Pi_H^a = E_H\left[\frac{\max(0, s-\bar{s}))}{R} - p + q(\bar{s})\right] \geq \Pi_L^a,\ \forall\ \bar{s} \in (V_{rec}, 1), \forall p, q(\bar{s}), R.$$

Moreover, there exists $\bar{s} \in (V_{rec}, 1)$ such that both equalities are strict.

units of a tranche with face value 1/x collateralized by one unit of the pool.

The proof of Lemma 1 immediately follows from the strict concavity (convexity) of Π_i^d (Π_i^a) at $\bar{s} \in (V_{rec}, 1)$, and the strict second-order stochastic dominance relationship of beliefs. It follows that type L agents are the natural buyers of collateralized debt, and H agents are the natural investors in the leveraged mortgage portfolio. In other words, if there is trade in collateralized debt in equilibrium $-b_H = b_L > 0$.

3.2 Equilibrium definition

Definition 1 *A general equilibrium is a set of prices* (p, $q(\overline{s})$) *and allocations* $\{c_i, c_i', a_i, b_i(\overline{s})\}_{i\in\{L,H\}}$ $\forall \overline{s}$*, such that both agents optimally choose their consumption and investments subject to their budget constraint and the collateral constraint* (2)*, the demand for the mortgage portfolio equals the fixed supply, and the collateralized debt market clears,*

$$b_H(\overline{s}) + b_L(\overline{s}) = 0, \ \forall \overline{s}.$$

3.3 Equilibrium characterization

Despite the simple nature of the environment, the equilibrium of the economy is complex because portfolios may include long and short positions in a continuum of debt contracts indexed by their face value $\bar{s}$. To simplify the equilibrium, and to capture the strong demand for senior tranches of RMBS and other securitizations before the post-2007 financial crisis, we assume that type L agents, who are the natural buyers of collateralized debt tranches, have a sufficiently high consumption endowment. We show how this assumption implies a pricing function of collateralized debt that substantially simplifies the characterization of the equilibrium. Moreover, in what follows, we normalize the discount factor such that $R = 1$.

Assumption A1 $n_L \geq E^s$*.*

Assumption A1 has two implications: first, the equilibrium price of a unit of the mortgage portfolio p is bounded below by the fundamental value E^s, since any lower price contradicts goods-market clearing in period 0, as it would give both types at least one investment possibility that they would strictly prefer over current consumption. Second, the total value of type L agents' endowment equals $n_L + p \geq 2E^s \geq 2max_{\overline{s}}\left[E_L[min\{s, \overline{s}\}]\right]$. So type L agents can afford to buy all collateralized debt at its maximum expected payoff. Since, moreover, they do not expect to make strictly positive profits from any other investment, they bid up the price of any collateralized debt issued by type H agents to their expected discounted value, where they are indifferent between investing and consuming, implying a debt price function

$$q(\overline{s}) = E_L[\min\{s, \overline{s}\}]. \tag{5}$$

As stated in Corollary 1, this implies that type H agents expect profits Π_H from buying the mortgage portfolio outright to be lower than from buying it using leverage, so we can focus on equilibria where type H agents leverage their entire mortgage portfolio holdings without loss of generality.[13]

Corollary 1 *With $q(\overline{s})$ given by (5) $\Pi_H^a \geq \Pi_H = E_H[s] - p$, $\forall p, \overline{s}$, with strict inequality for some $\overline{s} \in (V_{rec}, 1)$.*

3.3.1 Type H's problem and the choice of $\overline{s}$

The debt price function (5), together with Lemma 1 and Corollary 1, substantially simplify H investors' portfolio choice problem. This is because they imply that H investors expect to make strictly higher profits from investing in the mortgage pool, once it has been optimally leveraged with debt, than from any other investment. Their problem thus simplifies to choosing current consumption (which through the budget constraint determines their investment in the mortgage

[13]By requiring that type L agents be able to acquire collateralised debt of *any* face value $\overline{s}$ at their expected payoff, assumption A1 is stronger than what is needed in many cases. In particular, in most equilibria the face value $\overline{s}$ is much below its upper bound, such that smaller type L resources would also suffice to imply the bond-pricing function (5) for the relevant range of values of $\overline{s}$.

pool) and the level of leverage $\overline{s}$ given p and the price function $q(\overline{s})$:

$$\max_{c_H,\overline{s}} \quad U_H \quad = \quad c_H + (n_H + p - c_H)R^a_H. \tag{6}$$

where $R^a_H(p,\overline{s}) \doteq \frac{E^s - E_H(\min\{s,\overline{s}\})}{p - E_L\{\min\{s,\overline{s}\}\}}$ is the leveraged gross return of the mortgage pool using debt with riskiness $\overline{s}$.

The first-order condition for $\overline{s}$ can be written as:

$$\frac{(n_H + p - c_H)}{p - E_L\{min\{s,\overline{s}\}\}}[(1 - F_H(\overline{s})) - R^a_H(1 - F_L(\overline{s}))] = 0. \tag{7}$$

Equation (7) describes the tradeoff between higher debt repayments and increased funds for investment when choosing $\bar{s}$. In particular, a unit-increase in $\bar{s}$ increases H investors' expected discounted repayments by $1 - F_H(\overline{s})$, proportional to their perceived probability of paying back the full face value. It increases the expected returns by the increase in the price of collateralized debt (equal to $(1 - F_L(\overline{s})))$ multiplied by the expected return on investment R^a_H. Importantly, to the right of the single-crossing point E^s, H investors perceive a higher probability of full repayment than L investors $(1 - F_H(\bar{s}) \geq 1 - F_l(\bar{s}), \forall \bar{s} > E^s)$. A small increase in $\bar{s}$ thus increases H's expected discounted payments by more than the additional funds it raises from L investors. Nevertheless, according to (7), whenever H investors perceive the profitability of investing in the leveraged mortgage pool R^a_H to be higher than R, they find it optimal to raise $\bar{s}$ above E^s to raise additional funds for investment.

We concentrate on the case where the consumption endowment of H investors is large, and treat the more general case in Online Appendix A.3. In particular we concentrate on the case where investors are able to pay their reservation price for the asset, denoted $\bar{p}(\bar{s})$, that makes them indifferent between consuming and investing, defined as

$$\overline{p}(\bar{s}) \doteq E^s + E_L(min(s,\overline{s})) - E_H(min\{s,\overline{s}\}) \tag{8}$$

As we will see below, a sufficient condition for this is Assumption A2:

Assumption A2 $n_H > \underline{n_H} = E^s - E_L(min\{s, E^s\}) - E_H(min\{s, E^s\})$

3.3.2 The equilibrium price of mortgage collateral

The equilibrium in this economy is defined by two conditions: first, the optimal choice of leverage, or of face value $\bar{s}$, characterized by (7); and second, asset-market clearing for the mortgage pool. Proposition 1 shows how assumption A2 greatly simplifies the characterisation of equilibrium: because H investors are cash-rich, asset-market clearing requires p to equal the reservation price $\bar{p}(\bar{s})$. At their reservation price, however, H investors simply choose the face value $\bar{s}$ that maximises the difference in expected payments on debt, equal to E^s.[14]

Proposition 1 *A bubble in the price of mortgage collateral*

Under assumptions A1 and A2, there exists a unique equilibrium, with the following properties: the face value of debt $\bar{s}$ *is equal to* E^s*. The equilibrium price of the mortgage pool equals* $\bar{p}(E^s)$*, and thus strictly exceeds the common valuation of collateral cash-flow* E^s*. The price of collateralized debt* $q(\bar{s})$ *is given by* (5)*. Type* H *investors purchase the entire mortgage pool and use it as collateral for debt with face value* $\bar{s} = E^s$*. If this does not exhaust their first-period resources, they consume the rest. Similarly, type* L *agents purchase all collateralized debt, and consume any remaining available resources.*

Proof. Note that the single-crossing condition and (8) together imply that $\bar{p}(\bar{s})$ has a maximum at $\bar{s} = E^s$. Any price above $\bar{p}(E^s)$ thus reduces the demand for the mortgage pool to 0 because there is no $\bar{s}$ at which H investors do not strictly prefer to consume their endowment. If we can show that there is excess demand for the asset at any $p < \bar{p}(E^s)$, the proposition follows because $p = \bar{p}(E^s), \bar{s} = E^s$ is the unique solution to (7) and (8) and implies that investors are indifferent between consuming and investing and have sufficient funds to buy the mortgage pool.

[14]Proposition 3 in Appendix A.3 derives the unique pair $\bar{s}, p$ for general values of H investors' resources.

To see that there is excess demand for the asset at any $p < \bar{p}(E^s)$, note that, at any such price, H investors expect to make strictly positive profits from buying the mortgage pool using debt issued at $\bar{s} = E^s$. Moreover, at $p \leq \bar{p}(E^s)$, the optimal choice of $\bar{s}$ is never smaller than E^s according to (7). This implies that the resources H investors can raise through debt issuance are at least $2q(E^s)$. (5) and A2 then imply that total funds available for net investment in the mortgage pool are at least equal to $n_H + 2E_L(min\{s, E^s\}) > E^s + E_L(min\{s, E^s\}) - E_H(min\{s, E^s\}) = \bar{p}(E^s)$. The strict inequality implies that net demand for the asset by H investors exceeds the unit net supply from L investors at any $p < \bar{p}(E^s)$. ■

Note that Assumption A2 is sufficient but not necessary for there to be a bubble in the price of mortgage collateral. As long as there is some face value $\bar{s}$ such that type H investors can raise funds that suffice, together with their endowment n_H to purchase the asset at a price that exceeds E_s, there is a bubble in the asset price in equilibrium. Corollary 5 in Online Appendix A.3 discusses more general conditions for this.

3.4 Belief divergence and the price of mortgage collateral

This sub-section looks at the effect of 'belief divergence', defined as a further increase in the difference between θ_L and θ_H through infinitesimal changes $d\theta_L < 0$ and $d\theta_H > 0$.[15] By increasing the likelihood of high payoffs, $d\theta_H > 0$ increases type H's perceived upside potential of leveraged payoffs from the pool. And $d\theta_L < 0$ further reduces the riskiness of the collateralized debt as perceived by L types, and thus increases the price $q(\bar{s})$. For a given face value of the debt tranche $\bar{s}$, belief divergence thus raises type H's expected return on the leveraged mortgage portfolio, as well as her resources from debt issuance, which increases the equilibrium price of the mortgage pool p. Moreover, assumption A2 implies that the face value $\bar{s}$ remains unchanged, because the single-crossing point E^s is unaffected by diverging beliefs about dispersion around E^s.

[15] Again, the results in this section also hold with a general mean-preserving spread (contraction) in type H (L)'s beliefs about a generic stochastic payoff s. The results are available from the authors upon request.

Corollary 2 - *Under assumptions A1 and A2, belief divergence* $d\theta_L < 0, d\theta_H > 0$ *strictly increases the asset price* p.

Proof. The statement follows from $p = \overline{p}(E_s) \doteq E^s + E_L(min(s, E^s)) - E_H(min\{s, E^s\})$, the concavity (convexity) of $E_L(min(s, E^s))$ $(-E_H(min(s, E^s)))$ in $\bar{s}$, and the assumption that F_ε is strictly increasing. ■

Note that, again, Assumption A2 is sufficient but not necessary for the result. When Assumption A2 does not hold, equilibrium leverage $\bar{s}$ typically changes in response to changing beliefs. As long as the probability of full repayment is sufficiently flat around $\bar{s}$, however, $d\frac{1-F_i(\bar{s})}{d\theta_i}$ and the change of $\bar{s}$ in response to changing beliefs are small. In this case, belief divergence typically still increases the prices of debt and of the mortgage pool. It is, however, easy to construct counterexamples where $\frac{1-F_L(\bar{s})}{d\theta_L}$ is large enough to make belief divergence decrease prices. In particular, when L investors perceive payoff $\hat{s} > E^s$ to have high probability mass, such that F_L increases discretely at $\hat{s}$, H investors may find it optimal to choose a face value $\bar{s} = \hat{s}$. Whenever a reduction in risk perceived by L investors moves probability mass below $\hat{s}$, the equilibrium price of debt may fall sufficiently to decrease equilibrium collateral prices, as the following example shows (for the limit case of a discrete distribution).

Example 1 *Belief divergence may decrease asset prices*
For $V_{rec} = 0$, *consider the limit case of a discrete distribution on support* $\{0+a_i, 0.25+b_i, 0.75-b_i, 1-a_i\}, i \in \{H, L\}$ *with probability masses* $p_H = \{0.5, 0, 0, 0.5\}$ *and* $p_L = \{0.25, 0.25, 0.25, 0.25\}$. *Take* $R = 1$, $n_L = 0.5$, $n_H = 0.2$. *It is trivial to show that type* H *sets* $\overline{s} = 0.75 - b_L$ *for small enough* b_L *as the profit function has a kink at that value: raising* $\overline{s}$ *above* $0.75 - b_L$ *would increase her expected payments by more than the expected gain from higher price of collateralized debt,* $\overline{s} < 0.75 - b_L$ *would leave cheap debt unused. This implies* $q = 0.4375$ *and* $p = \overline{p} = 0.5625$ *when* $a_i = b_i = 0$. *Consider an increase in disagreement in form of small increases in either* b_L *or* a_L. *Since* $\frac{dp}{db_L} = -0.25$ *and* $\frac{dp}{da_L} = 0.25$, *belief divergence may increase or decrease the price of the loan portfolio.*

This section briefly illustrates the main differences of our benchmark results with respect to the case where there are optimist and pessimist investors who disagree about the mean payoff of the collateral pool, in the sense that the payoff distribution perceived by optimists dominates that of pessimists at first order (rather than at second order as in our benchmark analysis), as analyzed by Simsek (2013). Optimists expect payoffs from all (increasing) assets to be higher than pessimists. The collateral price thus crucially depends on optimists' resources, and is bounded by their optimistic valuation. In other words, there is never a bubble in the collateral price as in our benchmark analysis. Rather, when disagreement is concentrated on downside risk, the equilibrium asset price may equal the pessimistic valuation both with and without collateralization (as pointed out by Simsek (2013)).

3.5 Comparison with disagrement about mean payoffs

Consider a version of the environment with two alternative investor types that are pessimists and optimists, denoted P and O, respectively. Investors agree about the recovery value of the mortgage portfolio V_{rec}, but disagree about the mean payoff, in the sense that the distribution of payoffs perceived by optimists first-order stochastically dominates that perceived by pessimists, with strict inequality for all payoffs above the recovery value, such that $1-F_O(s) > 1-F_P(s) \forall s \in (V_{rec}, 1)$.

The following corollary assumes that, equivalent to Assumption A1, pessimists' endowments are sufficiently high: $n_P \geq E^s$. The bond price function (5) is then unchanged.

Corollary 3 *No bubble in collateral prices with disagreement about mean payoffs.*
With disagreement about the mean payoff, when $n_P \geq E^s$, there is no bubble in collateral prices. Moreover, the equilibrium price of collateral p is strictly lower than the optimistic valuation E^s_O when the recovery value V_{rec} or optimist resources are sufficiently low, such that $n_O < E^s_O - 2V_{rec}$ and optimists cannot fund the purchase of the mortgage portfolio using only own funds and riskless debt.

Proof. Note that the maximum price investors are willing to pay when issuing collateralized debt of face value $\bar{s}$ still equals $\overline{p}(\bar{s})$ in (8), with changed subscripts

$$\overline{p}(\bar{s}) \doteq E_O^s + E_P(min(s, \overline{s})) - E_O(min\{s, \overline{s}\}) \tag{9}$$

First-order stochastic dominance implies $E_P(min(s, \overline{s})) - E_O(min\{s, \overline{s}\}) \leq 0$, with strict inequality for $s \in (V_{rec}, 1]$, since the payments on collateralized loans that optimists expect to make are larger than the receipts expected by pessimists. Thus $\overline{p}(\bar{s}) \leq E_O^s$: the collateral price is bounded above by the optimist valuation. Whenever the sum of optimist endowment n_0 and funds raised by issuing riskless debt at $\bar{s} = V_{rec}$ does not suffice to buy the whole mortgage portfolio at $p = E_O^s$, the market price of collateral is bounded above by $max_{\bar{s} \in (V_{rec}, 1]} \overline{p}(\bar{s}) < E_O^s$.
■

The standard case of disagreement about mean payoffs does thus not imply a bubble in collateral prices, which remain bounded by the valuations of individual investors. But the possibility to use the asset as collateral for debt may increase its price by putting more funds into the hands of optimists.

Corollary 4 *Positive return to collateralization with disagreement about mean payoffs*
With disagreement about mean payoffs, the collateral price p may exceed the value of the mortgage portfolio sold without collateralization. In other words, there may be a positive return to collateralization.

The proof is by example.

Example 2 *Positive return to collateralization with disagreement about mean payoffs*
Consider $E_P = V_{rec}$, $E_O = 1$, $n_P = V_{rec}$, *and* $1 - 2V_{rec} < n_O < n_P$. *In this case, it is easy to see that the market price of collateral when collateralized loans are not traded equals* V_{rec}, *the pessimist valuation (since optimists cannot purchase pessimists' mortgage portfolio at any price higher than that). With collateralized loan trade, in contrast, optimists optimally buy pessimists'*

mortgage portfolio at a price of 1*, equal to their own optimistic valuation, which they can afford by issuing loans at the optimal face value* $\bar{s} = V_{rec}$.

Note the difference to our benchmark analysis: with disagreement about risk, a positive return to collateralization requires a bubble in asset prices, while with disagreement about mean payoffs it does not. A final example shows how the return to collateralization may be zero, and the collateral price equal to the pessimist valuation, when disagreement is concentrated on downside risks.[16] We choose a particularly stark example where optimists and pessimists only disagree about the probability of catastrophic losses (where payoffs are zero). In this case, the payments that optimists expect to make on any collateralised debt (of non-zero face value) exceed those that pessimists expect to receive by an amount equal to the total difference in expected payments from the mortgage pool. In other words, for optimists, a higher expected debt service exactly off-sets higher expected payoffs. This leaves their expected cash-flow from the leveraged asset equal to the pessimistic valuation of the mortgage pool.

Example 3 *No return to collateralization with disagreement about downside risk*
Consider a case where optimists and pessimists agree on the payoff distribution, but pessimists perceive a small probability $\mathbb{p}$ *of a catastrophic loss (where the mortgage pool pays zero), while optimists perceive no possibility of such a loss. Assume that optimists lack funds to buy the whole asset endowment of pessimists at the minimum price, equal to the pessimistic valuation of collateral cash-flow* E^s_P*. Because* $E_P(min(s, \overline{s})) - E_O(min\{s, \overline{s}\}) = E^s_P - E^s_O$ *in* (9)*, the price of the collateral is unaffected by collateralization and equal to the pessimistic valuation.*

[16] See Simsek (2013) for a more general discussion of disagreement about upside vs. downside risk.

4 Quantification: The effect of disagreement about risk on the Structured Finance boom

This section studies the quantitative effect of disagreement about risk on the price of structured debt securities such as the residential mortgage backed securities (RMBSs) backed by their tranches that experienced an unprecedented boom before the recent global financial crisis. We consider a version of the economy in Section 3 with $i = 1, .., I$ types of risk-neutral investors and homogeneous consumption endowment n. Collateral assets consist of a pool of $l = 1, ..., L$ mortgages of face value and mass 1. To keep the analysis tractable with many investors, we assume that the whole mortgage pool is endowed to a single originator who sells it in its entirety to investors in $t = 0$.

In $t = 1$ a stochastic fraction d of mortgages defaults and pays the recovery value $V_{rec} < 1$. We assume that investor i believes in the "market standard' (Morini, 2011, p. 127) model of credit risk before the crisis, the standard Gaussian copula model with homogeneous correlation (Li, 2000; Laurent and Gregory, 2005). She thus expects mortgage l to default whenever x_l, interpreted as the value of creditor l's assets, falls below a threshold $\overline{x}$ equal to the inverse standard normal distribution evaluated at the common default probability $\overline{\pi}$:

$$x_l = \rho_i \cdot M + \sqrt{1 - \rho_i^2} \cdot M_l < \overline{x} = \mathbb{N}^{-1}(\overline{\pi}), \;\; M, M_l \sim N(0, 1) \tag{10}$$

x_l equals the weighted average of an aggregate factor M, capturing economy-wide conditions, and a loan- or borrower-specific factor M_l, which are both distributed according to the standard normal distribution. As before, investors disagree about the importance of aggregate conditions in determining loan defaults, as summarized by the parameter ρ_i. Since ρ_i^2 equals the correlation between two individual creditors' asset values, investors with higher perceived ρ_i believe individual defaults to comove more strongly, and thus expect d to be less tightly distributed around $\overline{\pi}$. The normalization of the aggregate factor M to unit-variance implies that any disagreement about the volatility of aggregate conditions will be captured by the parameter ρ_i.

The disagreement about macroeconomic risk among forecasters in Section 2 is thus suggestive evidence for heterogeneous values of ρ_i. Together with the recovery value in case of default V_{rec}, ρ_i and $\overline{\pi}$ completely determine the distribution of the cash-flow from the mortgage pool equal to $s = 1 - d(1 - V_{rec})\ \forall d$.

The originator maximizes current profits from selling the loan pool to investors in one of two ways:[17] as shares in a 'pass-through' securitization that pays all investors their share in the total cash-flow that the collateral generates, equal to $1 - d(1 - V_{rec})\ \forall d$; or structured as an RMBS by splitting the cash-flow into 'tranches' that receive payments in strict order of their pre-specified seniority. Specifically, tranche 1 promises to make a total payment of $a_1 < 1$ to its holders in period 2, where a_1 is the 'detachment point' of tranche 1, and receives any cash-flow that defaulting and non-defaulting mortgages generate until a total of a_1 is reached. Tranche 2 promises to pay $a_2 - a_1$, where $a_1 < a_2 < 1$, but only receives cash-flow once a_1 has been paid to holders of the first tranche, etc.

Given one of the two securitization possibilities - structured or pass-through - an equilibrium is defined as a vector of prices such that the originator maximizes current profits, investors maximize utility, and the demand for all assets equals the supply.

4.1 Payoff distributions and valuation of tranches

To illustrate how investors' perceived correlation determines the perceived payoff distribution, we choose parameters to capture the characteristics of the market for US subprime mortgage-backed securities prior to the crisis. We consider RMBS consisting of 5000 mortgages (Coval et al., 2009a); a common perceived default probability $\overline{\pi}$ of 12.5 percent; and a recovery value V_{rec} that comoves inversely with the pool's default rate d in a range of $+/-15$ percentage points around $\overline{V}_{rec} = 60$ percent, reflecting longer time-until-foreclosure and lower resale values when

[17] After this paper had been complete, Ellis et al. (2017) considered the optimal security design in a similar framework.

default rates are high.[18] The analysis uses the same 6 tranche structure as Coval et al. (2009a): equity (100-97 percent), junior (97-93 percent), mezzanine I and II (93-88 and 88-80 percent respectively) and senior I and II (80 to 65 and 65 to 0 percent).

The left panel of Figure 3 shows how, for $\rho_i^2 = 0$, the perceived distribution of payoffs from an RMBS's collateral pool collapses around the expected payoff equal to $1 - \overline{\pi}(1 - \overline{V}_{rec}) = 95$ percent. As ρ_i^2 rises, the distribution fans out at a decreasing rate, but the lowest percentile remains above 75 percent as payoffs are protected by the recovery value.[19] The right panel of Figure 3 illustrates the sensitivity of the perceived payoff distribution to reducing the number of collateral assets to $L = 100$ and the recovery value to 0 (but keeping the default probability of loans equal to 12.5 percent). This makes the characteristics of the asset pool more similar to those of a typical CDO consisting of mezzanine RMBS tranches (Coval et al., 2009a).[20] The payoff distribution in the right panel is markedly different from the benchmark specification: diversification is less powerful with fewer collateral assets, such that even investors who perceive asset payoffs to be uncorrelated ($\rho_i^2 = 0$) perceive cash-flow risk. Moreover, for higher values of perceived correlation, the distribution has substantial mass at values as low as 60 percent (as payoffs are not protected by recovery values), and a positive probability of full repayment.

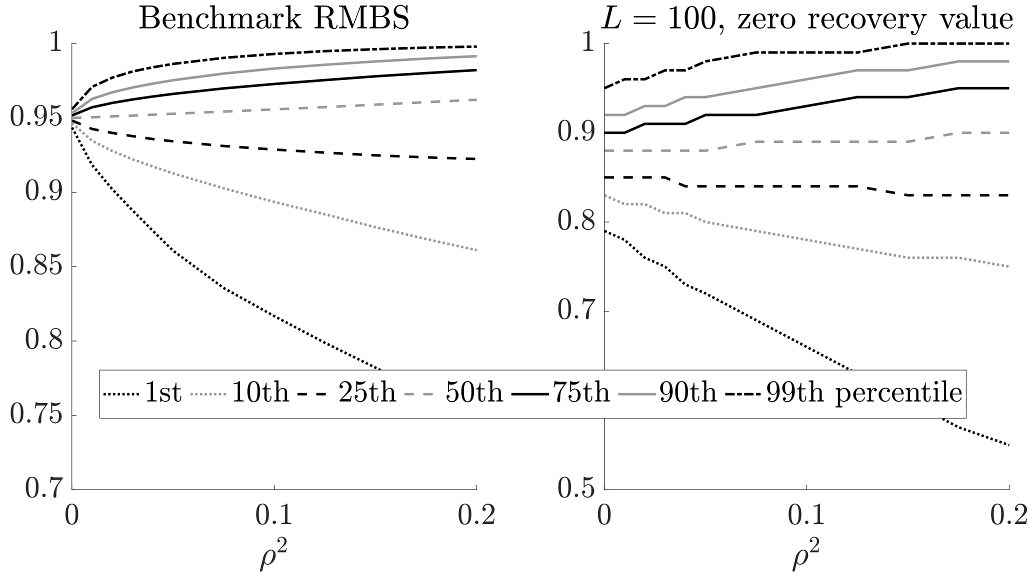
To illustrate how the heterogeneous perceived payoff distributions depicted in Figure 3 affect the expected payoffs of RMBS tranches, the left panel of Figure 4 shows the difference between their payoffs expected by an investor with perceived asset correlation ρ_i^2 (depicted along the bottom axes) and that expected by a 'zero correlation' investor (whose ρ_i^2 equals 0), as a percentage of the underlying collateral's face value (the 'width' of the tranche). As expected, the collateral value, or the total expected payoff from the mortgage pool (the starred dashed line

[18] The default rates for subprime mortgages differed strongly over time, fluctuating around 10 percent during the years of strong house price growth up to 2006 and increasing to above 40 percent thereafter (see, e.g., Beltran et al. (2013)). The recovery value equals $V_{rec} = 0.6 + (d - \overline{d})$, where $\overline{d}$ is the average default rate equal to $\overline{\pi}$, but is bounded by a minimum of 45 percent.

[19] To interpret the magnitudes, note that ρ_i^2 and $\widehat{\rho}_i^2$ do not equal default correlations. In fact, as Figure 3 in Broer (2018) shows, the correlation between default events of any two mortgages in the RMBS is about half as large as the correlation of the underlying asset value x_l.

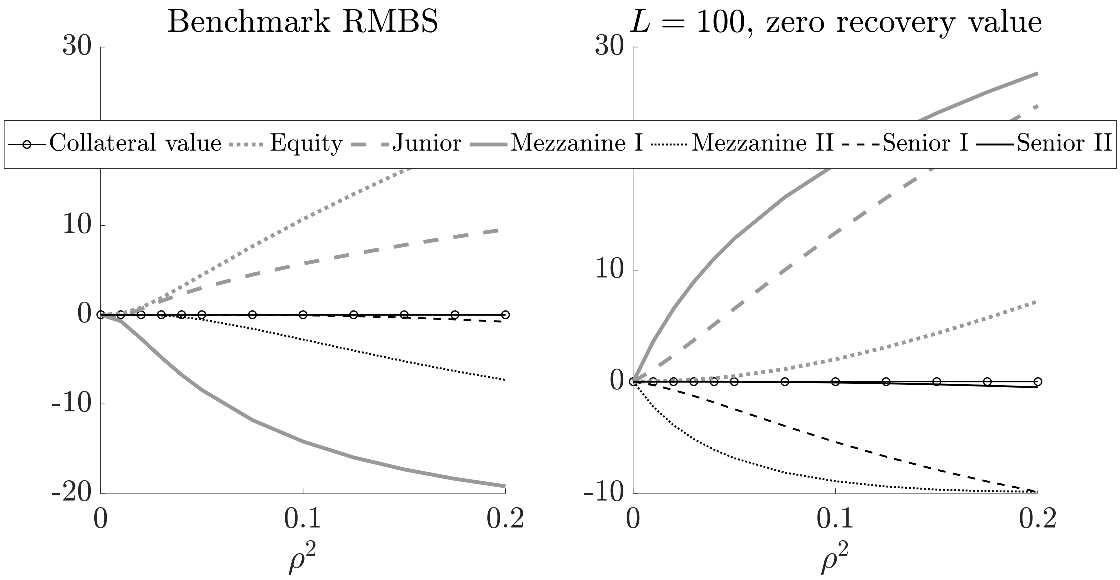
[20] See the working paper version of this article for an analysis of CDOs consisting of junior or mezzanine RMBS tranches, using the same modeling framework.

Figure 3: Distribution of collateral payoffs



The figure shows the distribution of collateral payoffs from two pools of mortgages: one with a high number of loans ($L = 5000$) and a substantial recovery value (averaging 60 percent, in the left panel); and a second with fewer loans ($L = 100$) and zero recovery value (right panel).

Figure 4: Expected value of tranches



For the two RMBSs in Figure 3, the figure shows the difference between their tranches' payoffs expected by an investor with perceived asset correlation ρ_i^2 (depicted along the bottom axes) and that expected by a 'zero correlation' investor (whose ρ_i^2 equals 0), as a percentage of the underlying collateral's face value (the 'width' of the tranche).

flat at 0), is unaffected by the perceptions of correlation as all investors share the same average default probability. Because the 'zero correlation' investor expects the payoff from the mortgage pool to equal 95 percent with certainty, she deems the junior and equity tranches of the RMBS, with attachment points close to or above 95 percent, to be worth nothing or little. High ρ_i^2 investors, in contrast, who perceive both a larger downside and upside risk, think that junior tranches are more likely to pay off, while they expect the mezzanine tranches to default with positive probability. Because even investors who perceive a high default correlation attach an extremely low probability to default rates of more than 40 percent, and because the recovery value of defaulting mortgage is about 50 percent, investors agree that senior RMBS tranches are (essentially) riskless.

The right panel of Figure 4 shows, again, the sensitivity to reducing the number of collateral assets to $L = 100$ and the recovery value to 0. The cash-flow is thus lower on average and more risky (in the right-hand panel of Figure 3). This has three implications: first, disagreement about expected tranche payoffs is strongest for more senior (here mezzanine) tranches; second, the differences in expected payoffs are larger (rising to almost 30 percent for mezzanine I tranches); and finally, these differences in expected values imply higher returns because the value of the more risky collateral pool, when sold as a pass-through securitisation, is lower. Thus, tranching more granular asset pools with higher risk (due to lower recovery values) is potentially more profitable for originators.

4.2 The return to tranching

Structuring a loan pool is profitable for originators whenever they can sell different tranches, but not the pool as a whole, to investors with high valuations. As Figure 4 shows, disagreement about default correlation creates strong incentives for structuring because it implies homogeneous valuations of the loan pool as a whole, but heterogeneous valuations of its tranches. With disagreement about mean payoffs, in contrast, tranching is powerful whenever optimists cannot afford the entire loan pool, as it allows originators to concentrate optimist demand on the particular ranges of the payoff distribution where disagreement is strongest. Figure 4 shows that this is relevant for RMBSs, as investors tend to agree on the risklessness of their senior tranches. The equilibrium return depends on the distribution of investors with different beliefs. This subsection briefly illustrates the equilibrium return to tranching in a simple numerical example for the subprime RMBS considered in the left panels of Figures 3 and 4, with five investor types that disagree about the default correlation ρ_i^2 or the default probability $\overline{\pi}_i$. We normalize the mortgage supply and the mass of each investor type to 1 and set the endowment n equal to $\frac{3}{7}$ for all investors, such that at least three types are needed to buy the benchmark mortgage pool.

Disagreement about default correlation

We assume a uniform distribution between two pairs of values $\{\rho_{min}^2, \rho_{max}^2\}$, corresponding to a 'weak' disagreement case (where $\{\rho_{min}^2$ and $\rho_{max}^2\}$ equal to 7 and 12 percent, respectively), and a 'strong' disagreement case ($\rho_{min}^2 = 2$ percent and $\rho_{max}^2 = 16$ percent).[21] The return to tranching, defined as the difference in market values between the benchmark RMBS and that of the collateral when sold as a non-tranched, pass-through securitization, equals 44 (111) basis points in the weak (strong) disagreement case. This return is high because self-selection is strong: disagreement about valuations is concentrated in the equity, junior and mezzanine tranches, which are cheap and can thus be bought by a small number of specialized investors with 'extreme' beliefs of high correlation (for junior and equity tranches), or low correlation (for mezzanine tranches). The remaining investors are happy to buy the senior tranches at their 'consensus' valuation.

[21] In the weak disagreement case, we choose a ρ_{min}^2 equal to the maximum correlation compatible with an AAA rating of both senior tranches in our benchmark RMBS. This is to capture the intuition that structures were designed to give senior tranches top credit ratings, and that rating agencies often had optimistic assessments of default probabilities (Griffin and Tang, 2011). In line with this, (Ashcraft et al., 2010, p. 13) find that the average fraction of subprime RMBS that received an AAA rating was 82 percent. We assume that the ρ_{max}^2-investors perceived a 0.5 percent probability of default rates in the RMBS pool reaching 40 percent or more, as observed in 2007 for US subprime mortgages (see, e.g., Beltran et al. (2013), especially figure 4). In the second, 'strong' disagreement specification, we extend the range of values such that the ρ_{min}^2-investor would also just give the mezzanine II tranche an AAA rating, and such that the ρ_{max}^2 investor perceives a probability of 1 percent of default rates rising to 40 percent or above.

Disagreement about average default probabilities

To illustrate the determinants of the return to tranching with disagreement about mean payoffs, consider the same RMBS with a homogeneous perceived correlation equal to 9.5 percent (the mid-point of the weak disagreement case above), but a distribution of investors across $\overline{\pi}_i$ that is uniform in a range $[10.5, 14.5]$ percent around the benchmark value of 12.5 percent (with a 5-point, equally spaced, support and investor endowments as before). Because investors agree that the senior tranches are riskless the issuer can sell them to pessimists at the common valuation. Equity, junior and mezzanine tranches, in contrast, can be sold to optimists. A pass-through securitisation, in contrast, requires three types to invest, and is thus priced by the median investor (with $\overline{\pi}_i = 12.5$ percent). The general equilibrium return to tranching thus equals the difference between the optimistic valuation and that of the median investor, in this case 113 basis points and thus approximately identical to that in the case of strong disagreement about default correlation.

4.3 Disagreement about risk and correlation trades

The results in this section suggest that both disagreement about risk and about mean payoffs may have been drivers of the surging demand for structured finance products before the financial crisis. One piece of evidence that supports the role of disagreement about risk, and thus complements the survey evidence in Section 2, is the popularity of "correlation trades" during the early 2000s. In those trades, speculators buy credit default swaps referencing CDO tranches in anticipation of their default, but offset the negative cash-flow from insurance premia with the returns on long positions in other tranches of the same (or an equivalent) securitisation. Depending on the perceived correlation of defaults in the pool of collateral assets, different kinds of such long-short trades are profitable. Investors who perceive a low default correlation purchase senior tranches (which they perceive as riskless) at the same time as insurance against the default of riskier ones (which they perceive to be junk). Investors who perceive volatile aggregate conditions, in contrast, find it profitable to buy protection against senior losses (which they perceive to be likely) and to insure, or buy, risky tranches (which they perceive to have upside potential). The former trading strategy was pursued, for example, by Morgan Stanley's Global Proprietary Credit Group, which suffered one of the biggest (and through Michael Lewis' bestseller "The Big Short" (Lewis, 2010) best-known) trading losses in financial history as the subprime crisis unfolded. The 'Magnetar Trade', in contrast, was hugely profitable for the US hedge fund of the same name that simultaneously invested in CDO equity tranches and in credit default swaps for more senior tranches (Mählmann, 2013). Indeed, these correlation trades are almost a unique feature of the structured-finance boom of the mid-2000s. With little trade before the early 2000s, volume surged in 2004 and 2005 (Corb (2012), p. 415), before falling back to insignificance after the financial crisis hit.

5 Conclusion

Motivated by the strong, and in the case of GDP forecasts rising disagreement about the dispersion of outcomes in US surveys of investors and forecasters, this paper has looked at the role of collateralized asset trade in economies where investors disagree about risk, rather than mean payoffs as in the literature. A simple static model of investor disagreement showed how the introduction of simple collateralized debt allows investors who perceive high payoff dispersion to purchase upside risk by investing in a mortgage pool and using it to collateralize debt tranches that their low-risk counterparts value highly. A quantitative application to the market of US subprime RMBSs showed how disagreement about the volatility of default rates, or the importance of aggregate factors for mortgage defaults, may substantially raise the price of junior RMBS tranches.

The theory presented in this paper has additional empirical predictions that can be compared to data even without information on the, typically unobserved, risk perceptions of investors.[22]

[22] A case of observed disagreement is that about ratings, as documented by Norden and Roscovan (2014) in a large sample of US and European firms. It would be interesting to study empirically how credit rating disagreement affects asset prices.

For example, our mechanism requires that investors can issue non-recourse collateralized loans. It thus predicts an effect of heterogeneous risk perceptions on the price of private-label residential mortgage-backed securities (but not of seemingly government-guaranteed agency securitizations), or on house prices in jurisdictions with non-recourse residential mortgages (but not, or less so, in those with recourse mortgages such as some US states and most European countries). Moreover, we would expect larger effects in markets where risk is important (in the sense of substantial default probabilities), and where disagreement about risk is likely to be stronger (such as for assets or contracts with a shorter history). Finally, we would expect the effect to increase over time both because disagreement about risk has seemingly increased (at least in the given sample of forecasters interviewed by the SPF), and because issuers of collateralized assets were increasingly able to draw on a more international and diverse investor pool. We leave formal empirical tests of these predictions to future research.

We also hope that our analysis opens some avenues for further theoretical research. We have abstracted from any additional dimensions of investor heterogeneity that may affect equilibrium asset prices. Importantly, it is sometimes argued that heterogeneity in risk appetite has encouraged the tranching of loan pools to create "safe" assets. In fact, Allen and Gale (1988) show how a debt-equity financial structure can increase the financial value of firms (relative to equity-only financing) when investors have heterogeneous risk aversion.[23] In addition, a dynamic analysis, where risk arises both from future payoffs and price movements, seems particularly interesting,[24] as do concrete applications of the theory to other financial markets. Finally, an investigation into the sources of disagreement, or the determinants of risk perceptions, would be valuable.

Our results imply that investors, on average, make losses relative to their required rate of

[23]Ellis et al. (2017) show how even with risk aversion the optimal equilibrium structure is such that tranches are sold to the investor with the maximum discounted expected payoff in equilibrium. They also show, however, that heterogeneous risk aversion as such may not encourage tranching (in an example with heterogeneous CARA preferences where heterogeneous portfolio shares of a risky asset imply homogenous valuations of any part of its cash-flow in equilibrium).

[24]The working paper version of this paper (Broer and Kero, 2014) presents a simple example of a dynamic equilibrium in a scenario with learning that tries to capture the main features of the Great Moderation in the US. As a subset of investors adjusts their posterior estimate of volatility more quickly to the Great Moderation than the rest, an increasing divergence of posteriors raises asset prices by between 5 and 20 percent.

return. This suggests that there might be welfare-improving policy interventions that would be interesting to study.[25] Moreover, the fact that disagreement about payoff dispersion makes investments more risky and raises leverage in the economy should be of interest for policy makers and regulators.

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Appendix for online publication only

A Additional Empirical Evidence

A.1 Disagreement in SPF forecasts - alternative measures

Our analysis of disagreement among SPF forecasters in Section 2.2 used a normal interpolation of forecasters' reported histograms. This allowed us to exactly decompose the measure of total disagreement into contributions of heterogeneous means and standard deviations. Figure 5 compares the disagreement measures in Figure 1 (dashed lines)) to an alternative without the normality assumption (solid lines). Specifically, the figure shows time series for the cross-sectional standard deviation of individual forecast dispersion (equal to the standard deviation of the forecast distribution), the cross-sectional standard deviation of forecast means, and the measure of total disagreement calculated as in (1), under both assumptions. All three measures are very similar to the benchmrk measures. Disagreement about the dispersion of GDP growth in the top panel is somewhat higher in levels than our benchmark measure, while total disagreement is somewhat lower. Importantly, however, the alternative measures correlate very strongly with those used for the main analysis.

A.2 Disagreement about US house price growth

This section complements the empirical analysis in the main document by quantitfying disagreement about the dispersion of future house prices among US home owners. This is important because one of the most common kinds of collateral for debt products is real estate. US private homes in particular collateralized a large fraction of the structured securitizations that experienced a huge boom-bust cycle around the recent financial crisis. This section briefly presents some evidence on disagreement about future growth in (average) US house prices. Unfortunately, data on house price expectations is not available for the period prior to the crisis, and most more

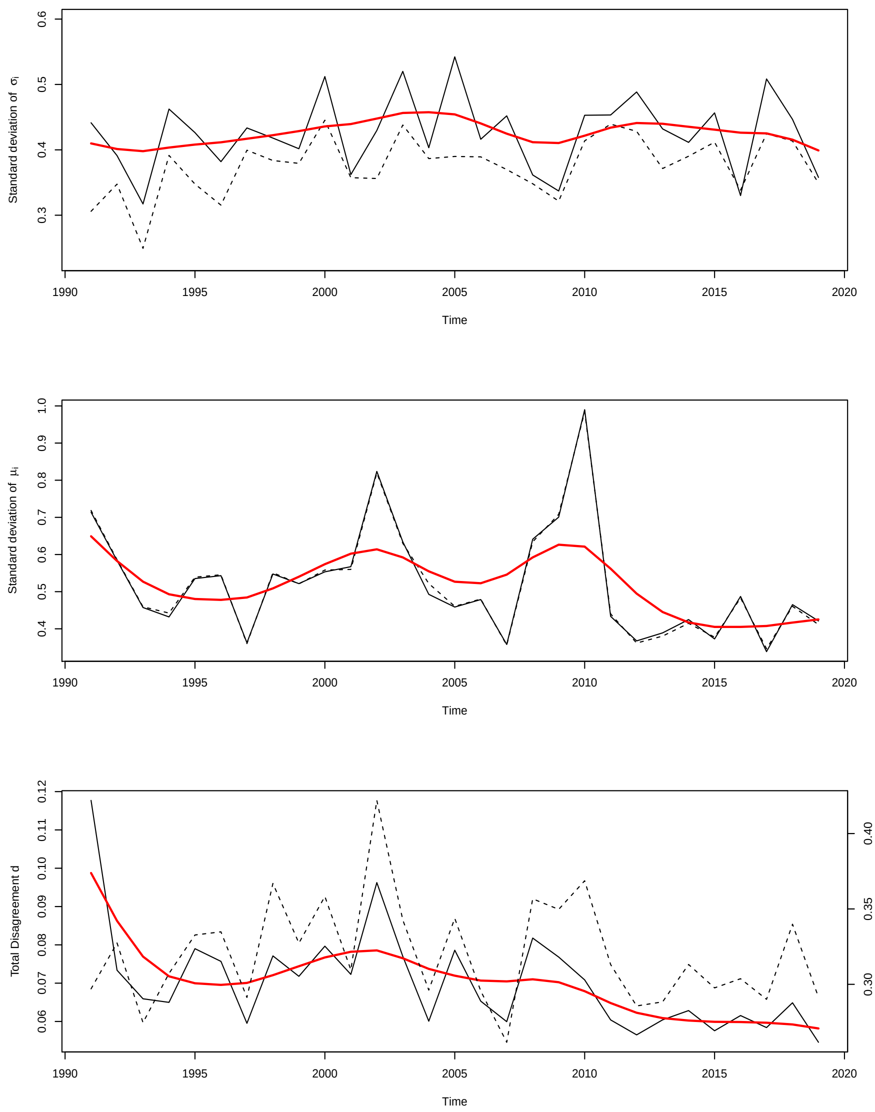

Figure 5: The top panel plots the time series of the standard deviations of forecast-standard deviations σ_{it} in the SPF using a normal approximation to the forecast distribution as in Giordani and Söderlind (2003) (dashed line), and an alternative series without that assumption (solid line). The center panel plots the corresponding standard deviations of means μ_{it}. The bottom panel shows the total disagreement measure d (center-left panel). The red lines in the panels show the trend in the alternative series using an HP filter with smoothing parameter 25 (to adjust for the annual frequency, see Ravn and Uhlig (2002)). We omit two observations at the beginning and end of the sample to reflect the two-sided nature of the filter.

recent surveys only ask respondents for their average expected price growth.[26] One exception is the Federal Reserve Bank of New York's monthly Survey of Consumer Expectations, whose respondents are asked to indicate a histogram of their perceived distribution of the growth in average US home prices over the following 12 months.[27] This data can be used to document the heterogeneity in home owners' house price expectations. For example, the average expected house price growth showed a slightly decreasing trend of around 4 percent during the sample period from June 2013 to December 2016.[28] At the same time, an average interquartile range of 3.9 percentage points indicates substantial heterogeneity in mean expectations. Importantly for this paper, the data also shows substantial heterogeneity in the perceived dispersion of house price growth: on average over the sample period 10 percent of respondents expect interquartile ranges larger than 6.4 percentage points, while another 10 percent expect them to be smaller than 1.1 percentage points. Using the same procedure as in Figure 1, heterogeneity in means and standard deviations contributed by almost exactly equal amounts to the overall disagreement about house price growth during the year ahead.

A.3 Equilibrium for general values of type H endowment n_H

Maintaining Assumption $A1$, this appendix considers the more general case where Assumption $A2$ does not hold. This implies, in particular, that the face value $\bar{s}$ may exceed E^s, and may change as the beliefs diverge.

Note that H investors take as given the price of the mortgage pool p, and the schedule for

[26] The monthly Michigan Survey of Consumer Sentiment's series of expected home price changes starts in March 2007. The quarterly Zillow / Pulsenomics survey of economists, investment strategists, and housing market analysts has an even shorter history.

[27] Source: Survey of Consumer Expectations, ©2013-2017 Bank of New York (FRBNY). The SCE data are available free of charge at http://www.newyorkfed.org/microeconomics/sce and may be used subject to license terms posted there. FRBNY disclaims any responsibility for this analysis and the interpretation of Survey of Consumer Expectations data. The exact wording of the question we focus on is: "And in your view, what would you say is the percent chance that, over the next 12 months, the average home price nationwide will increase / decrease by x % or more."

[28] We exclude respondents that do not own their primary residence, who only reported a point forecast, and those whose expected growth rate differed from that implied by their reported histogram by more than 5 percentage points.

the price of collateralised debt q as a function of $\bar{s}$ given by (5). If given p she prefers the mortgage pool over current consumption for some $\bar{s}$ (i.e. p is below her reservation price $p(\bar{s})$ for some $\bar{s}$), her problem is thus to choose $\bar{s}$ in order to maximise the return $R_H^a(p,\bar{s})$. In this case, proposition 2 shows that the first-order condition for $\bar{s}$ (7) has an interior solution.

Proposition 2 - *Interior choice of $\overline{s}$.*

Consider a given price p.

$$\text{If } \exists \overline{s} \in (V_{rec}, 1) : p \leq \overline{p}(\overline{s}). \tag{11}$$

then $R_H^a(p,\overline{s})$ has an interior maximum at some $\overline{s}^\star \in (V_{rec}, 1)$.

Proof of Proposition 2.

Note that p is bounded below by E^s from assumption A1. Also, $R_H^a(E^s,1) = 1$ by a limit argument. If $p = E^s$, then $R_H^a(E^s,V_{rec}) = R_H^a(E^s,1) = 1$ and condition (11) holds with strict inequality for $\overline{s} = E^s$, implying $R_H^a(E^s,E^s) > 1$. If $p > E^s$ then $R_H^a(p,V_{rec}) < 1$ and $R_H^a(p,1) < 1$. (11) implies that for some $\overline{s}' \in (V_{rec},1)$ $R_H^a(p,\overline{s}') \geq 1$. In both cases the statement then follows from continuity of R_H^a. ■

Proposition 3 shows that the equilibrium in this economy is defined by two conditions: first, the optimal choice of leverage $\overline{s}$; and second, asset market clearing for the mortgage pool, which defines the price such that H investors either exhaust all their wealth buying the entire mortgage pool, or are indifferent between investing and consuming. Intuitively, as the wealth of H investors rises, their increasing demand for the mortgage portfolio bids up the price until it reaches the indifference level $\overline{p}$.

Proposition 3 - *Existence and uniqueness of equilibrium.*

Denote as $\mathbb{B}(\overline{s}) = n_H + 2E_L[\min\{s,\overline{s}\}]$ the resources available to H investors for net purchases of mortgage collateral when they issue debt of face value $\bar{s}$ collateralized by the whole mortgage *portfolio in the economy. p and $\overline{s}$ are given by the unique solution of the following equations:*

$$\mathbb{C} \doteq (E^s - E_H[\min\{s, \overline{s}\}])(1 - F_L(\bar{s})) - (1 - F_H(\bar{s}))(p - E_L[min\{s, \overline{s}\}]) = 0, \quad (12)$$

$$p = min\{\overline{p}(\overline{s}), max\{E^s, \mathbb{B}(\overline{s})\}\}, \quad (13)$$

The price of collateralized debt $q(\overline{s})$ is given by (5). Type H investors purchase the entire mortgage pool and use it as collateral for debt with face value $\overline{s}$. If this does not exhaust their first-period resources, they consume the rest. Similarly, type L agents purchase all collateralized debt, and consume any remaining available resources.

Proof. (12) is simply the optimality condition (7) for choice of leverage $\bar{s}$. To understand (13), note that for any $p < E^s$ all agents would like to buy the mortgage pool, which cannot be an equilibrium. Equivalently, for any $p > \overline{p} \doteq E^s + E_L(min(s, \overline{s})) - E_H(min\{s, \overline{s}\})$ both type L and H investors would like to sell their endowment of the mortgage pool, again contradicting equilibrium. Type H optimality implies that they invest all resources in the pool using leverage when $E^s \leq p < \overline{p}$, but are indifferent between buying and consuming at $p = \overline{p}$. Thus, for $\overline{s}(\overline{p})$ the value of $\overline{s}$ that solves (12) when $p = \overline{p}$, if $\mathbb{B}(\overline{s}(\overline{p})) \geq \overline{p}$, type H's endowment is large enough to buy type L's endowment of the mortgage pool at the maximum price $\overline{p}$ that ensures her participation. There is thus an equilibrium price $\overline{p}$ at which H investors are happy to consume in period 0 any resources that remain after purchasing all of type L's endowment.

If for some price $p : E^s \leq p < \overline{p}$ it is true that $\mathbb{B}(\overline{s}(p)) < p$, H investors cannot buy the whole pool at that price but expect to make strictly positive profits $R_H^a > R$. Therefore they invest all their resources, equal to $\mathbb{B}(\overline{s})$, to buy type L's endowment. Noting that E^s is a lower bound for the equilibrium price as argued above gives (13).

Finally, to prove uniqueness, since $\mathbb{B}(\overline{s})$ is trivially strictly upward-sloping, it suffices to show that (12) is downward-sloping. This follows by totally differentiating (12)

$$\frac{dp}{d\overline{s}} = -\frac{\frac{d\mathbb{C}}{d\overline{s}}}{\frac{d\mathbb{C}}{d\overline{p}}} \quad (14)$$

Weak concavity of $R_H^a(\overline{s})$ at the optimum choice of $\overline{s}$ implies that the numerator is weakly negative. Since $\frac{d\mathbb{C}}{d\overline{p}} < 0, \forall p, \overline{s}$ the result follows. ■

Equation (13) states that the price of the mortgage pool is equal to $\mathbb{B}(\overline{s})$, the funds for investment available to H investors, but bounded by its fundamental value E^s below, and by H's maximal willingness to pay $\overline{p}(\overline{s})$ above. As in the case of cash-rich type H investors considered in the main text, when $p = \bar{p}(\bar{s})$, H investors optimally set $\bar{s} = E^s$ according to (7). This is because at $\bar{p}$, their expected return on investment R_H^a equals that which they have to pay to L investors. Thus, there are no gains from raising more funds by increasing $\bar{s}$ above the single-crossing point E^s where the difference in expected payments on collateralized debt is maximised.

Proposition 3 immediately implies that whenever type H investors have sufficient resources, there is a bubble in the equilibrium price of mortgage collateral p, defined as a price that exceeds the fundamental expected value of the mortgage pool E^s that is common to both investor types. Corollary 5 gives a condition for this that is more general than $A2$.

Corollary 5 *A bubble in collateral prices.*[29]
There is a bubble in the price of collateral assets, in the sense that the equilibrium price p strictly exceeds the fundamental value E^s, if the consumption endowment of H investors satisfies $n_H > \underline{n_H} \doteq E^s - 2E_L[\min\{s, \overline{s}\}]$.

Proof. If $n_H > E^s - 2E_L[\min\{s, \overline{s}\}]$, $\mathbb{B}(\overline{s}) > E^s$ and $\bar{p} > E^s$, so (13) implies the result. ■

Note that $\underline{n_H}$ may be negative, such that the existence of a bubble is independent of n_H. This is true, for example, when $F_\varepsilon(0) \leq \frac{1}{2}$, such that there is no "right-hand skew"' in F. [30]

[29] A previous working paper version of this article, Broer and Kero (2014), shows that similarly to the two-type economy, with heterogeneity in perceived risks across a continuum of types, the equilibrium price of the loan portfolio is necessarily above their common fundamental valuation. Unlike the two-type economy, however, these results require an exogenous upper bound for the face value $\bar{s}$.

[30] To see this, note that when $F_\varepsilon(0) \leq \frac{1}{2}$, (12) implies $(1 - F_L(\bar{s})) \leq (1 - F_H(\bar{s}))$, which holds only if the face value of debt weakly exceeds the single crossing point, such that $\bar{s} \geq E^s$. This implies that the resources from issuing collateralized debt equal $2E_L[\min\{s, \bar{s}\}] \geq 2E_L[\min\{s, E^s\}] \geq 2E^s(1 - F_L(E^s)) \geq 2E^s\frac{1}{2} \geq E^s$, implying $\mathbb{B}(\bar{s}) > E^s$, where the first inequality follows because $\bar{s} \geq E^s$ and $E_L[\min\{s, x\}]$ is increasing in x, the second inequality follows because payments on collateralized debt in states of full repayment are weakly smaller than total payments, and the third inequality imposes the assumption.

To understand the intuition for this, note that for a given expected payoff E^s, an increase in right-hand skew moves more probability mass below E^s (or probability mass below E^s to the left). This necessarily decreases the pay-offs of collateralised debt issued at face value $\overline{s} = E^s$. In the absence of right-hand skew, in contrast, the proceeds from issuing collateral debt at the optimal face value always suffice to drive the price of mortgage collateral above its fundamental value. So disagreement implies a bubble in collateral prices even when H investors have no own funds for investment because their consumption endowment equals 0.[31]

Whenever the equilibrium price is strictly between the fundamental value and its upper bound $\bar{p}$, any increase in H's endowment n_H drives up prices. This is stated in Corollary 6.

Corollary 6 *An increase in H's resources inflates the bubble.*
A rise in the endowment of type H investors n_H strictly increases the equilibrium price of collateral p when $\mathbb{B}(\overline{s}(E^s)) > E^s$ and $\mathbb{B}(\overline{s}(\bar{p})) < \bar{p}$.

[31] Proposition 6 in Ellis et al. (2017) makes a similar argument for a symmetric distribution.